

Pion Scattering Revisited

M. Ruiz-Altaba, J.L. Lucio* and M. Napsuciale*

*Departamento de Física Teórica, Instituto de Física
Universidad Nacional Autónoma de México, A.P. 20-364, 01000 México, D.F.*

**Instituto de Física, Universidad de Guanajuato
Loma del Bosque 103, 37160 León, Guanajuato, México*

Abstract. Chiral Ward identities lead to consistent accounting for the σ 's width in the linear sigma model's Feynman rules. Reanalysis of pion scattering data at threshold imply a mass for the σ of 600^{+200}_{-100} MeV.

This short talk (by M.R-A) reviews our recent work on the linear sigma model [1,2], where full references can be found. At low energies, chiral perturbation theory is supposed to yield good agreement with strong interaction data. Unfortunately, chiral perturbation theory gives rather poor results on the scattering lengths of pion-pion scattering, which are relevant experimental quantities in the limit of zero momentum, that is to say, where chiral perturbation theory should work best.

A missing ingredient in the description at low energies of strong interactions is the σ field, in addition to the Goldstone bosons of chiral symmetry (the pions). A wide scalar resonance in the vicinity of 600 MeV exists, and can be identified naturally with the σ particle of the original *linear* σ -model.

What are the phenomenological consequences of the linear σ -model in $\pi\pi \rightarrow \pi\pi$ scattering at very low energies? The sole guiding principle is chiral symmetry, whose Ward identities allow us modify the various vertices to take into account the large width of the σ resonance.

The chiral symmetry breaking giving mass to the pions is soft, so that when we include the width Γ_σ of the σ in its propagator, we can exploit the chiral Ward identities to modify the vertices accordingly. The chiral Ward identities are satisfied by the resulting lagrangian (with parameters m_π , f_π , m_σ), from which we compute the amplitudes in the various isospin and angular momentum channels of experimental relevance. We use the expression for Γ_σ from the decay $\sigma \rightarrow \pi\pi$ to perform a simple and succesful one-parameter (m_σ) fit to data.

The field σ is very unstable: its tree-level width is

$$\Gamma(\sigma \rightarrow \pi\pi) = \frac{3m_\sigma^3}{32\pi f_\pi^2}(1 - \epsilon)^2 \sqrt{1 - 4\epsilon}$$

where we have introduced the convenient shorthand $\varepsilon = (m_\pi/m_\sigma)^2$. In strict analogy with the Higgs field in the standard model, the σ width Γ_σ grows very fast with its mass: $\Gamma_\sigma(350) = 65$, $\Gamma_\sigma(500) = 310$, $\Gamma_\sigma(650) = 785$, all in MeV. The effect of the width of the σ field is to modify its propagator from the usual $i(q^2 - m_\sigma^2)^{-1}$ to $\Delta_\sigma(q) = i(q^2 - m_\sigma^2 + i\Gamma_\sigma m_\sigma \theta(q^2 - 4m_\pi^2))^{-1}$, where the step function ensures that the imaginary piece in the denominator appears only when the momentum of the propagator is above the kinematical threshold for σ decay.

Thus, in the physical process of $\pi\pi \rightarrow \pi\pi$ scattering, which we shall consider shortly, the propagator of the σ picks up the correction due to the width only in the s -channel, not in the u - nor the t -channels.

The crucial point is that, in the linear σ model, chiral symmetry is responsible for important cancellations which imply, notably, that the pion coupling is always derivative in the limit of soft pion momenta. Enforcing the chiral Ward identities on the vertices of the lagrangian implies that the latter pick up modifications related to the width Γ_σ . These vertex corrections depend on the kinematical variables (the incoming momenta) in a particular way, dictated by chiral symmetry. For instance, the $\sigma\pi^i\pi^j$ Feynman rule reads now

$$V_{\sigma\pi^i\pi^j} = \frac{-i}{f_\pi} \delta^{ij} (m_\sigma^2 - m_\pi^2 - i\Gamma_\sigma m_\sigma \theta(q^2 - 4m_\pi^2))$$

where q^μ is the momentum of the σ .

We find also

$$V_{\pi^i\pi^j\sigma\sigma} = V_{\sigma\sigma\sigma}\Delta_\sigma(p_j)V_{\sigma\pi^i\pi^j}$$

where p_j is the momentum of a pion, so that $p_j^2 = m_\pi^2$ if it is on-shell. This equation defines the $\pi\pi\sigma\sigma$ vertex. Similarly, the chiral Ward identity satisfied by the π^4 Feynman rule is

$$V_{\pi^i\pi^j\pi^k\pi^\ell} = V_{\pi^k\pi^\ell\sigma}\Delta_\sigma(p_j)V_{\sigma\pi^i\pi^j} + V_{\pi^i\pi^k\sigma}\Delta_\sigma(p_k)V_{\sigma\pi^j\pi^\ell} + V_{\pi^i\pi^\ell\sigma}\Delta_\sigma(p_\ell)V_{\sigma\pi^j\pi^k}$$

Obviously, these relations hold at tree level before chiral symmetry breaking, that is to say, when $m_\pi = 0$, and also $\Gamma_\sigma = 0$. Powerfully, they also hold when $m_\pi \neq 0$ and/or when $\Gamma_\sigma \neq 0$, to all orders in perturbation theory. This can be proved easily using the enormous advantage that the linear sigma model is a well-defined (renormalizable) field theory.

Since the vertex modifications ensure the preservation of exact chiral Ward identities, they also guarantee, for instance, that the pion couplings remain derivative as they should.

To illustrate the power of this implementation of chiral symmetry, we evaluate, at tree level, the amplitude for $\pi\pi$ scattering. Clearly, we do not expect the result to be the perfect answer, since the only resonance we will take into account is the σ . In particular, not taking into account the vector meson $\vec{\rho}^\mu$ is a rather bad approximation in the $I = 1$, $\ell = 1$ amplitude. Nevertheless, our results are in better agreement with experimental data than those of chiral perturbation theory. Let us

emphasize that the kinematical region where we compare both predictions, namely at very low momenta, is precisely where chiral perturbation theory should be exact. This lends further support to the real existence of σ as a strong resonance.

At tree level, four diagrams contribute to $\pi\pi \rightarrow \pi\pi$: the four-pion contact term, and the exchange of a σ in the three s , t and u channels. Due to the structure of the Feynman rules dictated by chiral Ward identities, the width Γ_σ contributes, in the Born approximation, only to $T_0^{(0)}$.

The experimental knowledge of pion scattering near threshold is rather poor. The relatively badly measured scattering lengths and ranges are $a_0^{(0)}$, $b_0^{(0)}$, $a_0^{(2)}$, $b_0^{(2)}$, $a_1^{(1)}$, $a_2^{(0)}$ and $a_2^{(2)}$. These seven numbers come out of our computation with only m_σ as a free parameter.

An overall fit to these seven numbers gives $m_\sigma = 700_{-150}^{+800}$ MeV. The χ^2 distribution is very flat towards increasing values of m_σ ; $m_\sigma \geq 550$ MeV is the only useful information.

Of the seven numbers, if we eliminate the worst one ($a_1^{(1)}$ (presumably under strong influence from ρ exchange, which we do not take into account)), then the fit improves and it yields $m_\sigma = 590_{-90}^{+220}$ MeV. Nicely, the fit to only the scalar isoscalar values gives $m_\sigma = 525_{-45}^{+85}$ MeV.

Overall, one may conclude that the data are consistent with a linear sigma resonance provided its mass is around 600 MeV (and thus its width also around 600 MeV). The errors on these numbers, from the pion data available, are substantial.

Although the low-energy moments $a_\ell^{(I)}$ and $b_\ell^{(I)}$ are the relevant quantities for us, what is actually measured is a momentum-dependent phase shift, which can be split in various isospin and angular momentum channels. From the analysis of the data available, we fit $m_\sigma = 550_{-80}^{+450}$ MeV. Again the error on the heavy side is huge: the χ^2 distribution is very flat with increasing m_σ .

Exact unitarity is achieved iff

$$\text{Im } T_\ell^{(I)} = \sqrt{\frac{s - 4m_\pi^2}{s}} |T_\ell^{(I)}|^2$$

from which the optical theorem can be derived. Since there are many other resonances in nature heavier than the σ , we should not worry much about possible unitarity violations at high momenta (say, above 1 GeV). It turns out that there is no problem with unitarity at center of mass momenta lower than the 400 MeV. Unfortunately, unitarity does not constrain m_σ from above in any meaningful way.

We have enhanced the linear sigma model by enforcing chiral Ward identities which take into account the (large) sigma width. We have found that low energy pion scattering data supports the existence of a wide σ field with mass around 600 MeV (actually $m_\sigma = 590_{-90}^{+220}$ MeV), provided we exclude the datum in the vector isovector channel. The advantage of keeping the σ as a true resonance in the effective low energy theory of strong interactions is not only that its inclusion simulates more or less the results of chiral perturbation theory to one loop, but

also, more crucially, that this opens the door to more industrious analyses of the whole scalar spectrum, including glueballs.

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REFERENCES

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